CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
MATHEMATICS FIELD DAY 2022

Team Medley, Junior Varsity Level

There are 6 problems. You will have 50 minutes. You should only write your solution and relevant work legibly on this paper. Use separate scratch paper for work that may not contribute meaningfully to your solution. When the time is up, you should convert and send your work as one pdf file. Only pdf files will be accepted. Each correct answer is worth ten points. Partial credit may be given only for substantial progess towards solution.

Calculators, cellphones, and other electronic devices are not allowed.

GOOD LUCK!

1. If $x + \dfrac{1}{x} = 1$, find the value of $x^3 + \dfrac{1}{x^3}$.

2. In triangle ABC, let D be the foot of the altitude from A. Prove that $AB^2+CD^2 = AC^2+BD^2$, where for example, AB means the length of the line segment joining points A and B.

3. Let ab_8 and ba_6 denote two-digit integer representations in base 8 and 6 respectively, where a and b are positive integers less than 6. If 5 is a common factor of ba_6 and $ab_8 - ba_6$, what is the value of ab_8 in base 10?

4. Consider a square whose side length is 2 inches long. If there are 5 points inside the square, prove that there exist at least one pair of the points such that the distance between the two points is less than $\sqrt{2}$ inches.

5. A point P is randomly selected from a triangular region whose vertices are $(0, 0)$, $(4a, 0)$, and $(4a, 3a)$, where a is a positive real number. What is the probability that P is at least a units away from each of vertices of the triangle?

6. In how many ways can the numbers $1, 2, 3, \ldots, 9$ be partitioned into disjoint sets, so that each set has a sum of 15? ("partitioned" means that each number has to be in one of the sets.)